Research Note: When is Versioning Optimal for Information Goods[1]?

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Abstract

This paper provides insights about when versioning is an optimal strategy for information goods. Our characterization of this class of goods is that variable costs are invariant with quality, including the special case of zero variable costs. Our analysis assumes a monopoly firm that has an existing product in the market and has an opportunity to segment the market by introducing additional lower-quality versions. We derive a simple decision rule for determining the optimality of versioning based on the solution to a single product maximization problem. Versioning is optimal when the optimal market share of the lower quality version, offered alone, is greater than the optimal market share of the high quality version, offered alone. A firm can profitably employ versioning for an information good if it can design the lower quality in a way that, relative to their valuations for the high-end version, high-type consumers have a lower relative valuation for the lower quality than do low-type consumers. When variable costs increase, a firm that offered only one product version need not consider adding another version. When variable costs decrease, the firm should explore adding a lower quality version.

Keywords: versioning, multiproduct monopoly, vertical differentiation, market segmentation, information goods

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Versioning—defining a product line with vertically differentiated quality levels—enables firms to endogenously create factors that segment consumers, thereby inducing them to self-select their preferred product variant. Versioning is recognized as a critical business strategy for information goods (Shapiro and Varian, 1998; Varian, 1998; Wei and Nault, 2005), and several researchers have demonstrated its usefulness in IT problems such as caching (Hosanagar, Krishnan, Chuang, and Choudhary, 2005), network externalities (Jing, 2007), intermediation (Bhargava and Choudhary, 2004), software piracy (Wu, Chen, and Anandalingam, 2003; Sundararajan, 2004a), music distribution (Snir, 2003), and software versioning (Ghose and Sundararajan, 2005; Raghunathan, 2000). Versioning is an attractive strategy for information goods such as software, music, movies and satellite images, because the firm can create low-quality variants at little additional cost. When should a seller of information goods adopt a versioning strategy? This paper examines this question, following the framework of Maskin and Riley (1984) and Deneckere and McAfee (1996) in which fixed costs of developing the highest quality are sunk, and the highest available quality is exogenously determined. This is justified by technological constraints, for example the picture quality and resolution of movies available on DVD is limited by DVD standards such as Blu-Ray and the resolution of commercially available satellite imagery is determined by available satellite technology and US government regulations.[1]

1 Model

Following Maskin and Riley (1984) and Deneckere and McAfee (1996), the pricing problem for a firm that has a high quality q_H and can offer additional lower quality versions from a feasible set

[1]Michael R. Hoversten, "U.S. national security and government regulation of commercial remote sensing from outer space," Air Force Law Review, Winter, 2001.

$\{q_1, q_2, \ldots q_{H-1}, q_H\}$, is

$$\max_{p_1 \ldots p_H} \quad \pi = \quad (p_H - c)(1 - F(v_H)) + \sum_{j=1}^{H-1} (p_j - c)(F(v_{j+1}) - F(v_j)) \tag{1}$$

$$\text{s.t.} \qquad U(v_j, q_j) - p_j = U(v_j, q_{j-1}) - p_{j-1} \qquad (\forall\ j = 1 \ldots H) \tag{2}$$

$$v_j \quad \geq \quad v_{j-1} \geq 0 \tag{3}$$

where consumer types v are arranged in increasing order ($U_v(v, q) > 0$), v indexes customer types, $F(v)$ is the cumulative distribution of customer types, $U(v, q)$ is type v's valuation for quality level q, p_j is price for q_j, and v_j is the type indifferent between the allocations (q_j, p_j) and (q_{j-1}, p_{j-1}). Because we focus on information goods, we model the variable costs (production, distribution etc.) as being invariant with quality, so that $c(q) = c$ for all q. This characterization is broader than the commonly used "zero marginal costs" assumption. For exposition, we write an arbitrary lower quality q_j as q_L, and we let $q_0 = 0$ with $U(v, q_0) = p_0 = 0$. We define demand elasticities $\epsilon(v; q_j)$, and $\eta(v; q_{j-1}, q_j)$ as:

$$\epsilon(v; q_j) \quad = \frac{\%\text{ change in sales of } q_j}{\%\text{ change in } p_j} \quad = \frac{U(v, q_j)}{U_v(v, q_j)} \left(\frac{F'(v)}{1 - F(v)} \right) \tag{4}$$

$$\eta(v; q_{j-1}, q_j) \quad = \frac{\%\text{ change in sales of } q_j}{\%\text{ change in } (p_j - p_{j-1})} \quad = \frac{U(v, q_j) - U(v, q_{j-1})}{U_v(v, q_j) - U_v(v, q_{j-1})} \left(\frac{F'(v)}{1 - F(v)} \right). \tag{5}$$

where $\eta(v; q_{j-1}, q_j)$ is the elasticity for q_j in the presence of lower quality q_{j-1}.

Assumption 1. For all feasible v such that $U(v, q_j) \geq c$, the function U is continuous and twice differentiable in v, with $U(v, q_j) > U(v, q_{j-1})$ (all types prefer higher quality) and $U_v(v, q_j) > U_v(v, q_{j-1})$ (higher types have higher marginal valuations for quality).

Assumption 2. $F(v)$ is continuous and monotonically increasing in v.

Assumption 3. Demand elasticities are increasing in price (equivalently they are increasing in

v), that is

$$\frac{\partial}{\partial v}\frac{U(v,q_j)}{U_v(v,q_j)}\left(\frac{F'(v)}{1-F(v)}\right) \geq 0 \tag{6}$$

$$\frac{\partial}{\partial v}\left(\frac{U(v,q_j)-U(v,q_{j-1})}{U_v(v,q_j)-U_v(v,q_{j-1})}\frac{F'(v)}{1-F(v)}\right) \geq 0 \tag{7}$$

These assumptions closely parallel those of Maskin and Riley (1984), we discuss the differences in §2.4.

2 Results

Answering the question "when is versioning optimal" involves establishing whether or not profit (Eq. 1) is optimized by selling q_H only. This can be viewed as a search in H-dimensional space, for optimal prices $p_1 \dots p_H$. Due to the constraints (Eq. 2–3) this is a problem of exponential complexity ($\mathcal{O}(N^H)$), and requires managers to know how consumers value each version and the cross-elasticity of demand across competing versions. Lemma 3 (in the Appendix) reduces the problem to $\mathcal{O}(N^2)$ complexity by showing that if none of the two-quality menus $\{q_i, q_H\}$ (for $i = 1 \dots H-1$) produces higher profit than q_H-only, then no menu with more than two quality levels does. Proposition 1 establishes a further reduction in complexity. To set up this result, let $\hat{p}_j$ denote the optimal price, and $\hat{v}_j$ the corresponding marginal buyer when version q_j is offered alone. Then, the optimal market share is $1 - F(\hat{v}_j)$, optimal price $\hat{p}_j = U(\hat{v}_j, q_j)$, and

$$\hat{v}_j = \begin{cases} \text{unique solution of } \left(\frac{U(v,q_j)-c}{U_v(v,q_j)}\frac{F'(v)}{1-F(v)} = 1\right) & \text{when} \frac{1}{\epsilon(0;q_j)} \geq \frac{U(0,q_j)-c}{U(0,q_j)} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

2.1 Versioning and Market Share

Proposition 1 *When q_H maximizes the firm's* optimal one-product market share*, it should be offered alone. Versioning is superior when the firm's optimal market share from selling some lower* *quality q_L alone exceeds that for q_H alone, for some $q_L \in (0, q_H)$. Mathematically, versioning is optimal when $\exists\ q_L : (1 - F(\hat{v}_L)) > (1 - F(\hat{v}_H))$.*

Proposition 1 reduces the complexity of the versioning decision for managers to that of determining optimal market share for a single version. This is a problem of $O(N)$ complexity, and requires no more information than that of the corresponding H single-product pricing problems. This result trivially explains prior findings that versioning is not optimal under linear valuations ($U(v, q) = v \cdot q$).

2.2 Versioning and Customer Heterogeneity

Next, consider the general intuition that versioning is attractive when there is substantial heterogeneity in marginal valuations for quality. Additional revenues from adding q_L to the product line should exceed the revenue loss due to cannibalization and information rent when high types have substantially greater marginal valuation for quality than do low types. But how much heterogeneity in marginal valuations is necessary to make versioning optimal?

Proposition 2 *For information goods with negligible variable costs, versioning is optimal when $\exists q_j \in (0, q_H)$ such that the* relative valuation $\frac{U(v,q_j)}{U(v,q_H)}$ *is decreasing in v. For $c > 0$, versioning is optimal when $\exists q_j \in (0, q_H)$ such that the* relative surplus from trade $\frac{U(v,q_j)-c}{U(v,q_H)-c}$ *is decreasing in v. Versioning is* not *optimal when $\frac{U(v,q_j)-c}{U(v,q_H)-c}$ is constant or increasing in v for every q_j.*

Proposition 2 shows that the choice of versioning strategy depends on how the low and high type consumers vary in their relative valuations for low and high quality products. The continuous q analog of this result is insightful: a single version is optimal when the relative change in profit margin ($\frac{U_v(v,q)}{U(v,q)-c}$) decreases with q, which implies that consumers are relatively homogeneous. Conversely, an increasing relative change in profit margin (increasing in q, sufficiently often) implies greater consumer heterogeneity, therefore it is optimal to version. Proposition 2 provides a precise quantifiable measure of the degree of consumer heterogeneity needed to make versioning optimal.

Proposition 2 can guide the implementation of a versioning strategy. Often, managers can intuitively determine whether relative valuation is lower for higher types. For example, managers understand that professional users of many software goods place disproportionately high value on security and network administration features, while novice users care little about them. Thus, if q_H represents the full version, and q_L a version that disables these features, then they can determine that the ratio of valuation for low type customers is close to 1 (its maximum value), while the ratio is much lower for high type customers, making it easy to conclude that selling both q_L and q_H will increase profits. Microsoft's strategy for selling the Windows XP operating system in 2001 exemplifies this point. Quoting from a computer industry website,[2] "*Windows XP comes in two different flavors with two different price tags: Home Edition and Professional Edition. Professional Edition packs in all of Home Edition's features, plus some corporate-strength capabilities that administrators and the security-conscious may want–for a $100 premium.*" Since the administration and security are extremely valuable to professional users while home users care little about them, home users have a higher relative valuation (for Home Edition divided by valuation for Professional Edition) than professional users. Other illustrations of this result are versions of tax preparation software (high-priced versions have features to handle capital gains) and instant messaging software (business versions have encrypted messaging).

2.3 Impact of Changes in Variable Costs

Once a firm has developed its highest quality product (q_H), introduced it to the market, and obtained a better sense of consumer preferences and heterogeneities, then any changes in demand or cost will necessitate a reexamination of the versioning decision. Moreover, because the firm cannot credibly commit to an existing product line, it may need to reexamine and reevaluate the versioning decision whenever it experiences changes in variable costs over time. Is such a reexamination necessary for every change in costs?

[2]CNET.com review, September 3, 2001, by Matt Lake. Available by request.

Corollary 1 Ceteris Paribus*, increasing the variable costs of the low and high quality version by an equal amount makes versioning less likely. Specifically, for any variable cost c at which versioning is optimal, there exists a cost ($c + \Delta c$, at which the optimal market shares from selling q_H-only and q_L-only are identical) such that versioning is no longer optimal.*

Corollary 1 also establishes that if a seller finds it optimal to version then versioning will remain optimal when variable costs decrease. A seller that finds it optimal to offer a single version will find that a single version remains optimal when variable costs increase. Therefore, whether or not a reexamination of the versioning strategy is necessary depends on the direction of change in variable costs as well as the existing versioning strategy. Cost changes are common in either direction — for example, the falling cost of IT equipment often leads to cost reductions, while labor and energy inputs often cause cost increases.

2.4 Connections with Prior Work

Proposition 2 has connections with prior results in the literature. Specializing the product space to two qualities $\{q_L, q_H\}$ and consumer utility functions to $U(v, q_H) = v$ and $U(v, q_L) = \lambda(v)$, yields Lemma 3 from Deneckere and McAfee (1996): *versioning is not optimal when* $\frac{\lambda(v)}{v}$ *is increasing in* v.[3] Specializing the utility functions even further to $U(v, q) = v \cdot q$ (constant marginal valuations for quality), yields a constant ratio $\frac{q_L}{q_H}$ implying that versioning is not optimal when $c = 0$, consistent with Bhargava and Choudhary (2001),Jones and Mendelson (1998) and Jing (2007) (under zero network effects). Proposition 2 generalizes the result: for $U(v, q) = r(v) \cdot s(q)$, the ratio of valuations is constant $\left(\frac{s(q_L)}{s(q_H)}\right)$ hence versioning is not optimal for all functions that are multiplicatively separable in consumer type and quality, covering functions that may be linear, concave or convex in quality. On the other hand, Proposition 2 also shows that seemingly minor variations in the

[3]On the other hand, our results apply in a class of scenarios ruled out by the Deneckere and McAfee (1996) assumptions. Consider $U(v, q) = v^2q^2 + vq$, with v being uniformly distributed in $[0, 1]$ and $q_L = 1, q_H = 2$. Conditions for applying Proposition 2 are satisfied whereas the corresponding transformation to the Deneckere and McAfee (1996) formulation violates necessary conditions (specifically, Eq. 10) for their results.

consumer valuation function cause divergence in outcome. For example, changing the function to $U(v, q) = vq + k$ (all users obtain some homogeneous benefit[4] k), reverses the result, and versioning is optimal.

While we have demonstrated the optimality of "one size fits all" for a large class of U functions, Maskin and Riley (1984) (hereafter referred to as MR84) show that complete versioning is usually optimal (see Proposition 4, Eq. 23, pp. 182, satisfied by most distributions). This difference in results can be explained by the differences in assumptions and the unique cost characteristics of information goods. One key difference is the treatment of variable costs. MR84 require $c(q) = c \cdot q$ (reasonable for traditional goods) while we focus on $c(q) = c$ (valid for most information goods because variable costs for information goods are nearly invariant with product quality). Due to this, our model yields a "one size fits all" outcome for some functions whereas, for the same functions, the MR84 $c(q) = c \cdot q$ produces a complete sorting result. A second difference is in the restrictions on U.[5] MR84 require $U_{qq} < 0$ (assumption 1) while we allow $U_{qq} > 0$; we also do not require the additional restriction on U stated in MR84 Assumption 4. This expands the space of feasible utility functions relative to MR84 and we have verified that versioning is not optimal for some of the functions[6] in this expanded set. Thus we find that "one size fits all" for a larger class of U functions in the case of information goods relative to the traditional goods analyzed in MR84. On the other hand, as demonstrated in the previous paragraph (example with $U(v, q) = vq + k$), the presence of positive network effects makes versioning more attractive in the case of information goods. Such network effects are quite commonly seen in software goods, community-oriented web sites, standards-based information services and hardware devices for viewing digital content.

[4]For example Katz and Shapiro (1985) model consumers as heterogenous in their basic willingness to pay for the product (v) but homogenous in the network benefit (k).

[5]MR84 adopt a continuous specification of quality levels while we employ a discrete quality space which simplifies the analysis without loss of generality (the increment between quality levels can be as small as desired). To compare the two formulations, we transform $p(q; v)$ in MR84 to $U_q(q, v)$. Then, the continuous q analog of our Assumption 1 is $U_q(v, q) > 0$ and $U_{vq}(v, q) > 0$, which is a subset of MR84 Assumption 1 part (i). Similarly, the continuous analog of our Assumption 3 is identical to MR84 Assumption 3. $U_q(q, v)$, $U_{vq}(q, v)$, and $U_{qq}(q, v)$ are derivatives of $U(q, v)$. A more detailed discussion of these connections is available from the authors.

[6]Examples: For $N > 1$, $F(v) = v$, and $c(q) = 0$: (1) $U(v, q) = q^N(q + v)$ and (2) $U(v, q) = v^N(q + v)$.

Note that our analysis, consistent with the referenced literature on versioning, has assumed that the fixed costs of creating lower quality versions are negligible. In practice, there may exist costs of segment management and product refinement. When these are materially significant, then the decision rule will need to be modified so that versioning is optimal only if the incremental profit from versioning exceeds the fixed costs of adding lower quality versions. Our analysis is limited to the domain of a monopoly seller, and it would be useful to examine its connections with issues related to competition and entry deterrence (e.g., Wei and Nault (2006)). Another topic for future research is the connection between our conclusions for quality-based price discrimination and the work on quantity-based price discrimination (Sundararajan, 2004b).

3 Conclusion

This article answers the question: when is versioning optimal for information goods? We show that versioning is profitable when the optimal market share of a lower quality version when offered alone exceeds the optimal market share of the high quality version offered alone. Our optimality condition simplifies the complexity of firms' decision on whether or not to version. We show that a firm can profitably employ versioning if its lower quality version gives high type consumers a lower percentage of their high-quality valuation, compared with the percentage valuation received by low type consumers. This insight may be useful to managers who have an intuitive sense about relative valuations for high and low type consumers, and can intuitively determine which product features may be removed from the high quality product to realize this property. Lastly, we show that variable costs impact the versioning decision so that firms may need to expand or shrink their product line as variable costs decrease or increase, respectively. A simple rule that informs managers when not to version is important as it can save time and effort that may otherwise be spent designing and pricing lower quality versions only to discover their sub-optimality.

Research on information goods has often focused on firms' strategy of offering multiple versions of information goods in many different contexts such as caching, network externalities, intermediation, software piracy, and music distribution. Our results contribute to this stream of research as they are general enough to be applied in many specific information goods contexts. Our results also apply to other physical goods that exhibit variable costs that are relatively invariant with quality, including (to give some examples from Deneckere and McAfee (1996)) printers, memory cards, vitamins, pocket calculators and media players.

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References

Bhargava, H. K., and V. Choudhary (2001): "Information Goods and Vertical Differentiation," *Journal of Management Information Systems*, 18(2), 85–102.

——— (2004): "Economics of Information Intermediaries with Aggregation Benefits," *Information Systems Research*, 15(1), 2236.

Deneckere, R. J., and R. P. McAfee (1996): "Damaged Goods," *Journal of Economics and Management Strategy*, 5(2), 149–174.

Ghose, A., and A. Sundararajan (2005): "Versioning and Quality Distortion in Software? Evidence from E-Commerce Panel Data," *NET Institute Working Paper No. 05-14*.

Hosanagar, K., R. Krishnan, J. Chuang, and V. Choudhary (2005): "Pricing and Resource Allocation in Caching Services with Multiple Levels of QoS," *Management Science*, 51(12), 1844–1859.

Jing, B. (2007): "Network Externalities and Market Segmentation in a Monopoly," *Economics Letters*, 95(1), 7–13.

Jones, R., and H. Mendelson (1998): "Product and Price Competition for Information Goods," Discussion paper, Stanford University.

Katz, M. L., and C. Shapiro (1985): "Network Externalities, Competition, and Compatibility," *American Economic Review*, 75, 424–440.

Maskin, E., and J. Riley (1984): "Monopoly with Incomplete Information," *Rand Journal of Economics*, 15(2), 171–194.

Raghunathan, S. (2000): "Software Editions: An Application of Segmentation Theory to the Packaged Software Market," *Journal of Management Information Systems*, 17(1), 87–114.

Shapiro, C., and H. Varian (1998): "Versioning: The Smart Way to Sell Information," *Harvard Business Review*, 76(6), 106–114.

Snir, E. M. (2003): "The Record Industry In An Era Of File Sharing: Lessons From Vertical Differentiation," in *Proceedings of the International Conference on Information Systems*, Seattle, WA.

Sundararajan, A. (2004a): "Managing Digital Piracy: Pricing and Protection," *Information Systems Research*, 15(3), 287308.

——— (2004b): "Nonlinear Pricing of Information Goods," *Management Science*, 50(12), 1660–1673.

Varian, H. R. (1998): "Versioning information goods," in *Internet Publishing and Beyond*, ed. by D. Hurley, B. Kahin, and H. R. Varian. MIT Press.

Wei, X. D., and B. R. Nault (2005): "Product Differentiation and Market Segmentation of Information Goods," *Haskayne School of Business Working Paper.*

——— (2006): "Vertically Differentiated Information Goods: Entry Deterrence, Rivalry Clear-out or Coexistence," *Conference on Information Systems and Technology.*

Wu, S., P. Chen, and G. Anandalingam (2003): "Fighting Information Goods Piracy With Versioning," in *Proceedings of the International Conference on Information Systems*, Seattle, WA.

A Technical Details

Propositions 1 and 2 require some intermediate results on two-quality versioning and the connection with N-quality versioning. Let Π_1^* be the profit from selling q_H alone, and $\Pi_2^*(q_L)$ the optimal profit under the two-quality menu $\{q_L, q_H\}$. A two-quality product line is optimal when there is some $q_L < q_H$ such that $\Pi_2^*(q_L) > \Pi_1^*$. Lemma 1 solves the $\{q_L, q_H\}$ pricing problem, and determines conditions for boundary vs interior solutions. Lemma 2 applies this result to obtain conditions for versioning (selling both qualities, cases A.2 and B.2) vs selling q_H only (cases A.1 and B.1). Finally, Lemma 3 generalizes the result to the case where the firm can choose from N quality levels, consisting of q_H and any set of lower quality levels. Let π^* be the optimal profit from considering all N quality levels. Lemma 3 proves that $(\pi^* = \pi_1^*) \Leftrightarrow (\pi_1^* = \pi_2^*(q_L)\ \forall q_L)$: thus versioning is (*is not*) an optimal strategy if and only if there exists (*does not exist*) a lower quality level q_L such that the two-quality menu $\{q_L, q_H\}$ is better than offering q_H alone ($\pi_2^*(q_L) > \pi_1^*$). To keep the notation concise, we replace $\epsilon(v; q_j)$ with $\epsilon(v)$, and $\eta(v; q_j, q_H)$ with $\eta(v)$.

A.1 Two-Quality Versioning Problem

The firm sets prices p_L and p_H for q_L and q_H respectively, and the individual rationality and incentive compatibility constraints yield

$$p_L = U(v_L, q_L) \tag{9}$$

$$p_H = p_L + U(v_H, q_H) - U(v_H, q_L) \tag{10}$$

The firm seeks to maximize profit

$$\Pi_2^*(q_L) = \max_{v_L, v_H} (1 - F(v_H))(U(v_H, q_H) - U(v_H, q_L)) + (1 - F(v_L))(U(v_L, q_L) - c)$$

subject to the constraints $0 \leq v_L \leq v_H \leq 1$. The first order conditions (FOC) can be expressed as

$$\frac{1}{\epsilon(v_L)} = \frac{U(v_L, q_L) - c}{U(v_L, q_L)} \tag{11}$$

$$\frac{1}{\eta(v_H)} = 1 \tag{12}$$

which may be interpreted as: *inverse elasticity of demand equals firm's market power* ($\frac{p-c}{p}$, also called the *Lerner Index*) *at the optimal prices.*

Lemma 1 *Eq. 11 yields a unique feasible solution* ($\tilde{v}_L \in [0, 1]$) *when* $\frac{1}{\epsilon(0)} \geq \frac{U(0,q_L)-c}{U(0,q_L)}$*, otherwise it generates a candidate solution at the boundary* $\tilde{v}_L = 0$*. Similarly, Eq. 12 yields a unique solution* ($\tilde{v}_H \in [0, 1]$) *when* $\frac{1}{\eta(0)} \geq 1$*, and a boundary solution* $\tilde{v}_H = 0$ *otherwise. The interior solution satisfies second-order conditions.*

Proof of Lemma 1. The term $\frac{U(v,q_L)-c}{U(v,q_L)}$ is strictly increasing with $v \in [0, 1]$, whereas $\frac{1}{\epsilon(v)}$ is decreasing in v (Assumption 4). Also $\frac{U(1,q_L)-c}{U(1,q_L)} > \frac{1}{\epsilon(1)}$ since the LHS approaches 1 while the

RHS approaches 0. Hence if $\frac{1}{\epsilon(0)} \geq \frac{U(0,q_L)-c}{U(0,q_L)}$, Eq. 11 yields exactly one critical point $\tilde{v}_L \in [0,1]$; otherwise, there is no interior solution and the optimal lies at the boundary $v_L = 0$ since the other boundary $v_L = 1$ is ruled out by $U(1, q_L) - c > 0$. A similar argument applies to $\tilde{v}_H$.

We have shown that there is a unique interior critical point $(\tilde{v}_L, \tilde{v}_H)$ when $\frac{1}{\epsilon(0)} \geq \frac{U(0,q_L)-c}{U(0,q_L)}$ and $\frac{1}{\eta(0)} > 1$. To confirm that the interior solution satisfies second-order conditions, we start by showing that the second derivative

$$\frac{\partial^2 \pi}{\partial v_L^2} = -F'(v)U_v(v, q_L) - F''(U(v, q_L) - c) + (1 - F(v))U_{vv}(v, q_L) - F'(v)U_v(v, q_L)$$

is negative at $\tilde{v}_L$. Differentiating $\epsilon(v)$ with v, Assumption 4 implies that

$$-U_v(v, q_L)F'(v) - U(v, q_L)F'' + \epsilon(v)[(1 - F(v))U_{vv}(v, q_L) - F'(v)U_v(v, q_L)] < 0$$

Substituting the FOC (Eq. 11) into this and rearranging the terms we get

$$F' \cdot c \cdot \frac{U_v(v, q_L)}{U(v, q_L)} + \frac{\partial^2 \pi}{\partial v_L^2} < 0$$

at $\tilde{v}_L$. Since F is strictly increasing, the first term is positive hence the condition requires that $\frac{\partial^2 \pi}{\partial v_L^2} < 0$. Similarly, $\frac{\partial^2 \pi}{\partial v_H^2} < 0$. Finally, the cross partial $\frac{\partial^2 \pi}{\partial v_H \partial v_L}$ vanishes, hence second-order conditions for maxima are satisfied. $\diamond$

Lemma 2 *For a specific two-quality menu $\{q_L, q_H\}$, versioning is optimal if and only if $\eta(\tilde{v}_L) < 1$. The optimal solution (v_L^*, v_H^*) is characterized by*

Case A *When $\frac{1}{\epsilon(0)} \leq \frac{U(0,q_L)-c}{U(0,q_L)}$ ($\equiv \tilde{v}_L = 0$) then*

1. *If $\frac{1}{\eta(0)} \leq 1$ then $v_H^* = v_L^* = 0$. Market is covered, but only q_H is sold.*
2. *If $\frac{1}{\eta(0)} > 1$ then $v_H^* = \tilde{v}_H > 0$, $v_L^* = 0$. Market is covered, versioning is optimal.*

Case B *When $\frac{1}{\epsilon(0)} > \frac{U(0,q_L)-c}{U(0,q_L)}$ ($\equiv \tilde{v}_L > 0$) then*

1. *If $\eta(\tilde{v}_L) \geq 1$ then versioning is not optimal, and*

$$v_L^* = v_H^* = \begin{cases} 0, & \text{if } \frac{U(v,q_H)-c}{U_v(v,q_H)} \frac{F'(v)}{1-F(v)} > 1 \\ \text{sol.}\left\{\frac{U(v,q_H)-c}{U_v(v,q_H)} \frac{F'(v)}{1-F(v)} = 1\right\}, & \text{otherwise} \end{cases}$$

2. *If $\eta(\tilde{v}_L) < 1$ then $v_H^* = \tilde{v}_H$, $v_L^* = \tilde{v}_L$, and $\tilde{v}_L < v_H^*$. Versioning is optimal.*

where $\tilde{v}_L$ and $\tilde{v}_H$ are defined in Lemma 1.

Proof of Lemma 2. For a specific two-quality menu $\{q_L, q_H\}$, the optimal solution (v_L^*, v_H^*) is characterized as follows.

Case A If $\frac{1}{\epsilon(0)} \leq \frac{U(0,q_L)-c}{U(0,q_L)}$, then $\tilde{v}_L = 0$ (by Lemma 1). Then

1. If $\frac{1}{\eta(0)} \leq 1$ then $\tilde{v}_H = 0$ due to violation of the two boundary constraints $v_L \geq 0$ and $v_H \geq 0$. Hence $v_H^* = v_L^* = 0$, only q_H is sold.
2. If $\frac{1}{\eta(0)} > 1$ then $v_H^* = \tilde{v}_H > 0$, $v_L^* = 0$ to satisfy the boundary $v_L \geq 0$.

Case B if $\frac{1}{\epsilon(0)} > \frac{U(0,q_L)-c}{U(0,q_L)}$, then $\tilde{v}_L > 0$ (by Lemma 1). Then

1. Since $\eta(v)$ is an increasing function, Equation 12 yields a solution $\tilde{v}_H \leq \tilde{v}_L$ if and only if $\eta(\tilde{v}_L) \geq 1$. Since this violates the boundary $v_H \geq v_L$, therefore $v_H^* = v_L^*$ (versioning is not optimal) is the solution to the q_H-only optimization problem. This problem has a boundary solution $v_H^* = 0$ when $\frac{U(v,q_H)-c}{U_v(v,q_H)} \frac{F'(v)}{1-F(v)} > 1$ at $v = 0$, otherwise $v_H^* = sol.\left\{\frac{U(v,q_H)-c}{U_v(v,q_H)} \frac{F'(v)}{1-F(v)} = 1\right\}$.
2. As in the previous case, since $\eta(v)$ is increasing, we get $\tilde{v}_H > \tilde{v}_L$ when $\eta(\tilde{v}_L) < 1$. Hence $v_H^* = \tilde{v}_H > v_L^* = \tilde{v}_L$. Versioning is optimal.

Hence proved. ◇

A.2 The H Quality Versioning Problem

Lemma 3 *Let $q_1, ..., q_H$ denote H quality levels. A multi-quality menu is optimal if and only if there exists a two-quality menu $\{q_i, q_H\}$ that yields greater profit than selling only q_H. Formally, if π^*, π_1^*, π_2^* are as defined above, then*

$$(\pi^* = \pi_1^*) \Leftrightarrow (\pi_1^* = \pi_2^*(q_i) \ \forall q_i)$$

Proof of Lemma 3. Our approach to proving the result is to show that whenever the optimal product line is $\{q_i, q_j, q_H\}$ (for some q_i and q_j, $i < j$) then offering $\{q_j, q_H\}$ is superior to offering $\{q_H\}$ only. When the optimal product line has H (rather than 3) qualities, the proof generalizes in a straightforward manner but the algebra is more tedious.

Therefore, let us suppose that $\{q_i, q_j, q_H\}$ is optimal. In other words, the optimal pricing problem for given quality levels $\{q_i, q_j, q_H\}$ and consumer valuation function $U(v, q)$ defined over these qualities, has an interior optimal solution $v_i^* < v_j^* < v_H^*$ (it is strictly interior, except for the possibility that $v_i^* = 0$, which in any case does not change the essential argument) where the v's are the respective indifference points. Formulating the optimization problem, and denoting the profit $\Pi(\{q_i, q_j, q_H\})$ as Π_3 we get

$$\begin{aligned}\Pi_3 = &(1 - F(v_H))(U(v_H, q_H) - U(v_H, q_j)) \\ &+ (1 - F(v_j))(U(v_j, q_j) - U(v_j, q_i)) \\ &+ (1 - F(v_i))(U(v_i, q_i) - c)\end{aligned}$$

Writing the hazard rate function $\frac{f(v)}{1-F(v)}$ as $\tau(v)$, Assumption 4 implies that each of the following terms is increasing in v: $\frac{U(v,q_H)-U(v,q_j)}{U_v(v,q_H)-U_v(v,q_j)}\tau(v)$, $\frac{U(v,q_j)-U(v,q_i)}{U_v(v,q_j)-U_v(v,q_i)}\tau(v)$, and $\frac{U(v,q_i)}{U_v(v,q_i)}\tau(v)$. The first-order conditions for this problem tell us that the optimal solution (v_H^*, v_j^*, v_i^*) is obtained where $\frac{U(v,q_H)-U(v,q_j)}{U_v(v,q_H)-U_v(v,q_j)}\tau(v) = 1$, $\frac{U(v,q_j)-U(v,q_i)}{U_v(v,q_j)-U_v(v,q_i)}\tau(v) = 1$, and $\frac{U(v,q_i)-c}{U_v(v,q_i)}\tau(v) = 1$ respectively.

Since we know that $v_j^* < v_H^*$ and since the term $\frac{U(v,q_H)-U(v,q_j)}{U_v(v,q_H)-U_v(v,q_j)}\tau(v)$ is increasing in v, this implies that $\frac{U(v,q_H)-U(v,q_j)}{U_v(v,q_H)-U_v(v,q_j)}\tau(v)$ is strictly less than 1 at v_j^*. A similar argument follows for the term $\frac{U(v,q_i)-c}{U_v(v,q_i)}\tau(v)$ evaluated at v_j^*. Note that $\frac{U(v,q_i)-c}{U_v(v,q_i)}\tau(v) = 1$ at a unique point v_i^* and is equal to infinity at $v = 1$ therefore it must be greater than one everywhere in $(v_i^*, 1)$. Therefore, the following inequalities hold at v_j^*

$$\frac{U(v,q_H)-U(v,q_j)}{U_v(v,q_H)-U_v(v,q_j)}\tau(v) \quad < \quad \frac{U(v,q_j)-U(v,q_i)}{U_v(v,q_j)-U_v(v,q_i)}\tau(v) \tag{13}$$

$$\frac{U(v,q_j)-U(v,q_i)}{U_v(v,q_j)-U_v(v,q_i)}\tau(v) \quad < \quad \frac{U(v,q_i)-c}{U_v(v,q_i)}\tau(v) \tag{14}$$

From our assumptions ($U_q(v,q) \geq 0$ and $U_{vq}(v,q) \geq 0$), all terms above, including the differences, are non-negative. Hence we can rewrite Eq. 14 as

$$\frac{U(v,q_j)-U(v,q_i)}{U_v(v,q_j)-U_v(v,q_i)}\tau(v) < \frac{U(v,q_j)-c}{U_v(v,q_j)}\tau(v) \tag{15}$$

Combining Eq. 13 and Eq. 15 the following inequality holds at v_j^*

$$\frac{U(v,q_H)-U(v,q_j)}{U_v(v,q_H)-U_v(v,q_j)}\tau(v) < \frac{U(v,q_j)-c}{U_v(v,q_j)}\tau(v) \tag{16}$$

Now consider the problem with two quality levels $\{q_j, q_H\}$ and let $(\tilde{v}_H^*, \tilde{v}_j^*)$ denote the optimal indifference points for the pricing problem. Solving the optimization problem we see that Eq. 16 guarantees $\tilde{v}_H^* > \tilde{v}_j^*$, and it is optimal to offer both qualities. Hence we have shown that whenever it is optimal to offer 3 quality levels, there exists a two-quality product line that outperforms offering q_H only. $\diamond$

A.3 Proof of Propositions 1 and 2

Eq. 8 and Eq. 11 imply that $\hat{v}_L = \tilde{v}_L$, i.e., the marginal consumer for q_L is identical whether it is offered alone or in conjunction with q_H. Further, from Lemma 2 (and recalling that $\hat{v}_j$ is the optimal indifference point for a single-quality (q_j) optimization problem),

$$\text{versioning is optimal if and only if } \eta(v) < 1 \text{ at } \hat{v}_L = \tilde{v}_L. \tag{17}$$

We prove below the equivalence of Eq. 17 with the conditions of Propositions 1 and 2.

1. **Proof of Proposition 1.**

The claim to be proved is that versioning is optimal if and only if $\exists q_L : \hat{v}_L < \hat{v}_H$. Following Eq. 17 this entails showing that $\eta(\hat{v}_L) < 1$ iff $\hat{v}_L < \hat{v}_H$. First, rewrite $(\eta(\hat{v}_L) < 1)$ as

$$\underbrace{\left[\frac{U(v,q_H) - U(v,q_L)}{U_v(v,q_H) - U_v(v,q_L)} \frac{F'(v)}{1-F(v)}\right]_{v=\hat{v}_L}}_{\eta(\hat{v}_L)\ldots Eq.\ 7} < \underbrace{\left[\frac{U(v,q_L) - c}{U_v(v,q_L)} \frac{F'(v)}{1-F(v)}\right]_{v=\hat{v}_L}}_{=1\ldots Eq.\ 8} \tag{18}$$

Second, we rewrite $\hat{v}_L < \hat{v}_H$ by applying the following transformations to Eq. 8. By definition, the LHS of Eq. 8 is 1 at $\hat{v}_L$ if we plug in $q = q_L$. Similarly, it equals 1 at $\hat{v}_H$ for $q = q_H$, therefore it must be less than 1 for $v < \hat{v}_H$ (because the LHS is increasing in v; from Assumption 4). Therefore, $(\hat{v}_L < \hat{v}_H)$ if and only if

$$\underbrace{\left[\frac{U(v,q_H) - c}{U_v(v,q_H)} \frac{F'(v)}{1-F(v)}\right]_{v=\hat{v}_L}} < \underbrace{\left[\frac{U(v,q_L) - c}{U_v(v,q_L)} \frac{F'(v)}{1-F(v)}\right]_{v=\hat{v}_L}}_{=1\ldots Eq.\ 8} \tag{19}$$

Applying the algebraic condition $\left(\frac{a}{b} < \frac{c}{d}\right) \equiv \left(\frac{a-c}{b-d} < \frac{c}{d}\right)$ (which holds when $a, b, c, d > 0$), establishes that Eq. 18 and Eq. 19 are identical.

2. **Proof of Proposition 2.**

   The statement "relative surplus from trade is decreasing in v" in Proposition 2 is mathematically expressed as: $\frac{\partial}{\partial v}\left(\frac{U(v,q_L)-c}{U(v,q_H)-c}\right) < 0$. Computing this derivative and multiplying both sides with $\frac{F'(v)}{1-F(v)}$ yields

$$\frac{U(v,q_H) - c}{U_v(v,q_H)} \frac{F'(v)}{1-F(v)} < \frac{U(v,q_L) - c}{U_v(v,q_L)} \frac{F'(v)}{1-F(v)} \quad \text{for all } v.$$

   Applying the condition at $\hat{v}_L$ generates a condition which is identical to Eq. 19. Similar argument applies to the case where relative surplus is *increasing* in v. Note that Proposition 2 makes a weaker statement in terms of sufficient conditions for optimality and non-optimality of versioning (rather than necessary and sufficient conditions).