
Comparison of Software Quality Under Perpetual Licensing and Software as a Service

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ABSTRACT: Software is available through a number of different licensing models such as the commonly used perpetual licensing model and a relatively new licensing model called *software as a service* (SaaS). There are several differences between SaaS and perpetual licensing. SaaS licensing offers software using a subscription model, whereas perpetual licensing involves a one-time payment for a perpetual use license and optional additional payments for future upgrades. Prior literature has not considered the impact of these licensing schemes on the publisher's incentive to invest in software quality. We model differences in how new software features are disseminated in SaaS and perpetual licensing. We show that these differences affect the publisher's incentive to invest in product development. We find that the SaaS licensing model leads to greater investment in product development under most conditions. This increased investment leads to higher software quality in equilibrium under SaaS as compared to perpetual licensing. The software publisher earns greater profits and social welfare is higher under SaaS under these conditions.

KEY WORDS AND PHRASES: application service providers, information goods, monopoly, on-demand computing, pricing, product development, software as a service, software licensing, software quality.

SOFTWARE AS A SERVICE (SAAS) IS A RAPIDLY GROWING means of software licensing. Many large and small software publishers are planning to embrace SaaS in the near future and it is expected to grow at the rate of 50 percent per year [29]. SaaS is different from traditional software licensing, which involves the buyer's purchasing a perpetual

use license from the software publisher and then making additional investments for hardware, installation, and maintenance. In contrast, in the SaaS model, users buy a subscription to the software and the software publisher (seller) runs and maintains the software on his own hardware. Users with current subscriptions can obtain access to the software using the Internet. Early successful adopters of the SaaS model include firms such as Salesforce.com and NetSuite (netsuite.com). Salesforce.com offers on-demand customer relationship management software solutions built on its infrastructure and delivered directly to users over the Internet. It has annual revenues of over \$309 million. Salesforce.com does not sell perpetual licenses; instead, it charges a monthly subscription fee starting at \$65/user/month. Similarly, NetSuite offers subscription-based access to its enterprise resource planning (ERP) software, which is targeted toward small and medium-sized businesses. Smaller businesses are less willing to invest in large, expensive systems that they have to maintain and are thus attracted to the SaaS model.

The trade press has used several terms to refer to SaaS, including *on-demand computing*, *seamless computing*, and *adaptive computing*. A number of articles in the trade press have painted a positive picture of industry adoption and growth of the SaaS model. The *Economist* claims that “SaaS is quicker, easier and cheaper to deploy than traditional software, which means technology budgets can be focused on providing competitive advantage, rather than maintenance” [29, p. 61]. Other trade publications are equally optimistic about SaaS; for example, the title of a recent International Data Corporation (IDC) white paper on SaaS reads: “The Future of Software Licensing: Software Licensing Under Siege” [20]. Credit Suisse First Boston has released an index to track this sector, and their analyst John Maynard claims that “traditional software is already dead” [29, p. 61].

SaaS has evolved from prior attempts to rent software and have it hosted by a third party. Choudhary et al. [8] list several technologies that enable software renting where users could use needed software for short periods of time and pay based on usage time. This was followed by the application service provider (ASP) model where the user could purchase a perpetual license but execute the software on hardware owned by a third party (ASP). Thus the user could outsource the hardware and infrastructure maintenance to an ASP. The ASP model is different from SaaS in several aspects. Whereas SaaS involves subscription payments to the publisher, the ASP model requires an up-front license fee payment to the publisher and a subscription fee to the ASP. The firms providing ASP services usually lack application-specific expertise and instead focus on providing reliable and secure access to their servers as their core competency. The ASP vendor does not add new features to the application, and software upgrades must be purchased from the publisher and paid for by the user. Thus the incentive effects of the SaaS model examined in this paper do not arise in the context of ASP.

SaaS differs from traditional software in several aspects. Trade journals point out that SaaS does not require large up-front investments and thus there is an impact on cash flows of both buyer and seller with small, stable cash flows under SaaS rather than large periodic payments. Data security is another issue because users' data are stored on the vendors' hardware and systems. The University of Florida lists predict-

able costs, increased bargaining power, ability to switch across providers, and up-to-date software as the key benefits based on a one-year experiment with SaaS [12]. In this paper, we focus on the last of these reported benefits—that is, users' access to the latest and most current version of the software. The publisher's choice of the SaaS versus the perpetual licensing model is typically considered to be a marketing decision that follows product development. In contrast, we show that these licensing decisions influence the firm's incentive to invest in product development. In particular, we focus on how software publishers' incentive to invest in product development differs between traditional software development and SaaS. The difference in incentives arises due to an SaaS vendor's incentive to make available the most current features in the software solutions.

New features become available to users at different times depending on whether the publisher adopts the SaaS model or perpetual licensing. Under the perpetual licensing model, new features are typically made available as part of a new version or upgrade. Publishers also provide product updates in the form of "patches." These patches are usually designed to repair features that were promised in the existing software and to protect against security vulnerabilities that arise from improper code. Typically, patches do not offer substantial new features. The reason is that under perpetual licensing, software publishers earn revenues from future upgrades whereas software patches are made available at no additional cost. For example, Microsoft released Office 2007 to consumers on January 30, 2007. The professional version costs \$499 for a new purchase and \$329 for an upgrade.¹ If Microsoft were to provide the new features included in Office 2007 as patches to users of Office 2003, the users may not be willing to upgrade or may only be willing to pay much less than \$329 for the upgrade. Thus a profit-maximizing publisher has an incentive to provide new substantial features as part of an upgrade that is priced appropriately. In contrast, the SaaS model provides access to new features as soon as they are available due to the subscription pricing model. Arora et al. [1] analyze the timing of product release for a monopolist who issues patches. In their model, the number of bugs in the software is an exogenously determined function of time. They compare a physical goods monopolist who incurs a marginal cost to patch each unit to a software monopolist who incurs a fixed cost for each patch. They find that a software monopolist releases the product sooner and with more bugs but invests more in patching compared to a physical goods monopolist. However, compared to the socially efficient solution, a software monopolist releases the product later with fewer bugs but invests less in patching. Kim et al. [19] examine the role of liability and software patching quality in determining the software quality of a monopoly vendor. Rahman et al. [25] examine a software publisher's incentive to enforce antipiracy measures by limiting the distribution of patches to legitimate users.

There is extensive literature on pricing strategies for information goods (e.g., [2, 7, 13, 17, 22, 30]). Bhargava and Choudhary [3, 4] examine conditions for the optimality of versioning for information goods. Clemons et al. [9] argue that firms can use information to develop more customized products, and the increased ease of access to product information enhances the value of product differentiation. Dewan and Freimer

[10] study software vendors' strategy of bundling software add-ins together with the base product and find that such a strategy can increase profits and social welfare while reducing prices. Dewan et al. [11] analyze online product customization and show that a monopolist seller may find it optimal to sell standard products in addition to customized products. In a duopoly model, they find that when the competitors simultaneously adopt customization technology, their standard products become less differentiated but the price of the standard products is unchanged. Kauffman and Wang [18] study online dynamic pricing at a group-buying Web site. They find three effects—a positive participation externality effect; buyers' expectation of a price drop (price drop effect); and an ending effect, where a lot of orders were placed in the last three hours of the auction. Lang and Vragov [21] compare pricing policies and profits for content providers when they can choose either a client-server model or a peer-to-peer (P2P) model, finding that content providers can achieve higher profits with the P2P model. Sankaranarayanan [26] analyzes the commitment problem of a monopoly in the context of information goods. Sundararajan [28] analyzes fixed-fee and usage-based pricing of information goods and finds that the addition of fixed-fee pricing is profit maximizing in the presence of positive transactions costs.

Academic literature on SaaS is limited with most articles focused on optimal pricing strategies under a pay-per-use model of software. Motivated by the emergence of the on-demand computing environment, Bhargava and Sundaresan [5] analyze contingent auctions showing how different levels of commitment can affect prices and revenues. Gurnani and Karlapalem [14] examine a vendor's strategy of supplementing perpetual use license with pay-per-use and report that this can be a profitable strategy for the vendor. Haruvy et al. [15] examine the role of piracy in affecting the adoption of subscription software products using an individual-level model in an adaptive population. Huang and Sundararajan [16] examine pricing strategies in a pay-per-use on-demand computing environment. They analyze the effect of four different factors—cost of deploying information technology (IT) in-house, the business value of IT, the scale of the provider's infrastructure, and the variable costs of providing service. Seidmann and Ma [27] examine a firm's choice between buying and maintaining a software application versus buying services on a per transaction basis from an ASP. They find that the ASP will serve users with light demand whereas high-demand users will prefer to buy the software. Zhang and Seidmann [31] examine the selling and renting of software and show that software vendors can price discriminate and segment the market by using subscription and licensing strategies together.

A key limitation of the aforementioned literature on software renting and ASP pricing is that it assumes that software quality is exogenously determined. This assumption precludes an examination of the impact of various licensing schemes on optimal software quality. In contrast, we compare perpetual software licensing to SaaS with a focus on the publisher's incentive to invest in software quality. To understand the key difference modeled in this paper, consider a software publisher that sells perpetual software licenses such as Microsoft. In the operating systems (OS) market, Microsoft is a near-monopoly and it releases OS software periodically. Some of its OS releases include Windows 95, Windows 98, Windows 2000, Windows XP, and the

recently released Windows Vista. From a user perspective, the purchase of a perpetual license represents substantial investment and becomes sunk cost once the purchase is completed. When a new version of the software becomes available, users who have previously purchased the software have a choice between continuing to use existing software (at no additional cost) or spending more money to purchase the new version. Users do not upgrade unless the new software provides substantial incremental benefits relative to the previous version. For instance, although Microsoft has released newer versions since Windows 98, a large number of users have not upgraded. Microsoft tried to end support for Windows 98 in January 2004 but the public outcry following that announcement forced Microsoft to reschedule the cutoff date to June 2006 [6]. Thus users upgrade to a newer version only if there is sufficient benefit to them that exceeds the cost of upgrading. It took Microsoft more than five years to develop Windows Vista and the new features in it. However, because all of these features are bundled together in Vista, users had to wait for the release of the completed OS. Thus features that Microsoft developers may have finished in 2002 were not available to end users until the complete OS was released in 2007.

In contrast to the perpetual licensing model, which relies on sales of perpetual licenses and upgrades, the SaaS model utilizes the subscription model. This alleviates some of the problems with perpetual licensing described above. There is no competition between the present and future versions of software because the vendor does not sell a perpetual use license. Thus the publisher does not need to hold back new features for the next version and subscribers may have a higher willingness to pay because they expect to receive further enhancements to software features. SaaS alters a vendor's incentive to invest in software development by allowing the vendor to release new features as soon as they are finished² and make them available to all subscribers. Thus a key advantage offered by SaaS is that individual features can be released as soon as they are completed, whereas the perpetual licensing model requires them to be withheld until a new version of the software is completed. In this paper, we argue that this little-noticed property of SaaS to deliver faster time to market for new features provides SaaS with a significant advantage over perpetual licensing. In particular, we focus on how this affects the vendor's incentive to invest in software development. Given the convex cost of developing software quality, it is reasonable to expect that the vendor will reduce his software development budget and yet have higher "average" quality due to the faster time to market. This would lead to lower equilibrium (final) quality but higher profits for SaaS relative to perpetual licensing. However, counter to this expectation, our model shows that, in most cases, the publisher will invest more in software development under SaaS and thus have higher equilibrium quality.

Model

WE WISH TO DETERMINE WHETHER THERE ARE DIFFERENCES in the optimal level of investment in product development for a software publisher under two different licensing schemes—the traditional way of licensing with perpetual licenses and the emerging paradigm of SaaS where software is provided using a subscription model. Differences

in the optimal level of investment translate into differences in resulting software quality that can affect optimal prices, profitability, and social welfare. We compare the software quality provided by the publisher when the firm offers a standard “perpetual use” license as compared to the quality when the firm offers “software as a service.”

Following substantial literature on product quality (e.g., [23]), we model quality as a vertical attribute where all users prefer higher quality to lower quality holding everything else constant (such as price).

We develop a two-period model where the vendor invests in product development to improve product quality. Thus product quality is a continuous nondecreasing function of time. For example, software publishers such as Microsoft, Oracle, and Salesforce.com are continually making improvements to their software. In our model, the trajectory of the firm’s quality is assumed to be linear:

$$q(t) = s \cdot t,$$

where $q(t)$ is quality as a function of time (t), and s is the rate of change of quality over time (slope). The assumption of linear quality trajectory is a simplification. This trajectory can be changed by the firm by hiring or firing employees or through the use of overtime. However, in our stylized model, the firm has full information at the beginning of the game and can therefore determine the trajectory at that time. At time $t = 0$, the firm decides how much to invest in quality, thus determining the trajectory (slope) of its software quality over the two periods. At time $t = 1$, it releases the product to the market and continues software development. At $t = 2$, the firm releases the upgraded version of the software product (in the case of perpetual licensing), achieving a final quality $q(t = 2) = q_2$. At $t = 2$, the vendor ceases product development. (See Table 1.)

The quality of the software product at $t = 1$ is $q(t = 1) = s \cdot 1 = q_1$. Hence,

$$s = q_1.$$

Further, the quality of the product at $t = 2$ is $q(t = 2) = s \cdot 2 = 2 \cdot q_1$:

$$q_2 = 2(q_1). \quad (1)$$

The firm incurs fixed costs that are a convex function of the slope of the product improvement curve (q_1): $c \cdot q_1^2$.

Note that the cost function can also be equivalently expressed in terms of the final quality q_2 . Thus we can use Equation (1) to write the fixed cost of quality as

$$c \cdot q_1^2 = c \cdot (q_2/2)^2 = \hat{c} \cdot q_2^2,$$

where $\hat{c} = c/4$. Thus the fixed costs are costs incurred for product development in the first two periods.

The cost of quality can be different under the two licensing schemes. In particular, the cost of quality can be greater for SaaS if it becomes increasingly difficult to make quality improvements incrementally. For example, it may be necessary to periodically develop a new design and architecture for the software requiring the publisher to discard and redevelop parts of the software. While the same process occurs for perpetual licensing, this may temporarily disrupt the incremental development model of SaaS

Table 1. Summary of Key Notation

Symbol	Explanation
p_1, p_2, p_u	Price in period 1, price in period 2, upgrade price
q_1, q_2, q_p, q_s	Software quality at different time periods and under different licensing schemes
$\theta; \theta_u, \theta_1, \theta_2, \theta_n$	Buyer type; Various indifferent buyer types
μ	Ratio of second-generation users to the number of first-generation users
c	Scaling parameter for fixed cost of quality
ω	Ratio of cost of quality for SaaS relative to perpetual licensing
λ	Benefit from basic features, component of utility function
SW_p, SW_s	Social welfare under perpetual licensing, social welfare under SaaS licensing

and could potentially raise the cost of quality for SaaS. We allow for this possibility by including a parameter ω and model the fixed cost of quality for SaaS as $\omega \cdot c \cdot q_1^2$. Note that $\omega > 1$ implies a higher cost of quality for SaaS relative to perpetual licensing.

It is important to understand differences in the way users experience quality under perpetual licensing and SaaS. Under perpetual licensing, users in period 1 ($1 \leq t < 2$) experience q_1 and those who buy/upgrade in period 2 ($2 \leq t < 3$) experience q_2 . Under SaaS, users experience $q(t)$, which is linearly increasing until $t = 2$ and flat thereafter (see Figure 1). This is consistent with anecdotal evidence and our previous description of Microsoft's OS release schedule versus the way Salesforce.com continually upgrades its product for subscribers. Product development ceases at $t = 2$ ($q(t)$ is flat for $t > 2$) so that SaaS and perpetual licensing have the same time period for product development. When using perpetual licensing, the publisher has no incentive to continue product development beyond $t = 2$ because this is the last period of the game and the publisher will have no opportunity to release the next version. In contrast, the SaaS publishers can continue product development because users experience quality as the function $q(t)$. However, this would give an advantage to the SaaS model and make it difficult to compare SaaS quality with perpetual licensing due to their different development time periods. In order to retain comparability, we assume that the SaaS vendor stops product development at the same time as the vendor with perpetual licensing at time $t = 2$.

At the beginning of the first period, the firm reveals its product with quality q_1 . All consumers are fully informed about the product's quality trajectory. The publisher also declares his prices for periods 1 and 2. We assume that the publisher can credibly commit to second-period prices using one of many commitment devices listed in prior literature such as a money-back guarantee. Consumers then decide whether or not to purchase the software and, if purchasing, then the timing of the purchase.

Users' willingness to pay for the software product is a function of the quality of the software that is made available to users and the time period over which they are able to use it. As stated previously, we use a vertical differentiation model of quality where buyers are heterogeneous in their "taste" for quality (θ) with different buyers willing to

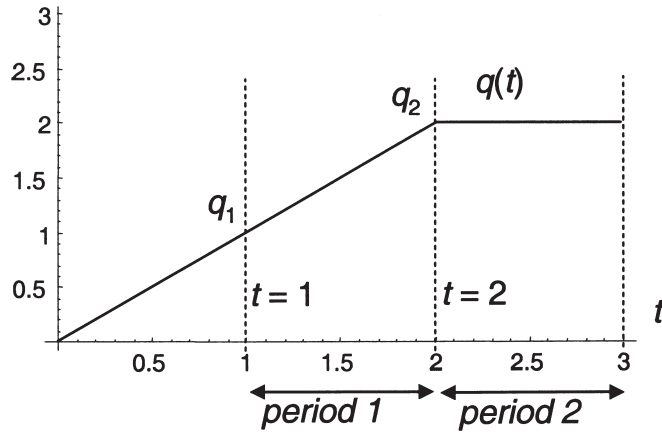


Figure 1. Product Quality Over Time

Note: Under perpetual licensing, users experience q_1 and q_2 , whereas under SaaS, users experience quality as a function of time $q(t)$.

pay different amounts for a unit improvement in software quality. Rearranging buyer types in increasing order and indexing by θ , we can write the buyer’s willingness to pay for each period of usage under perpetual licensing as

$$U(\theta, q) = \theta \cdot q_i + \lambda \cdot q_1, \tag{2a}$$

where $q_i \in \{q_1, q_2\}$ is the quality experienced by the buyer— q_1 in period 1, and q_2 in period 2, and $\lambda \in (0, 1)$ is a scaling constant. Thus, first-period utility for q_1 is $\theta \cdot q_1 + \lambda \cdot q_1$ and second-period utility for q_2 is $\theta \cdot q_2 + \lambda \cdot q_1$.

Similarly, the buyer’s willingness to pay for an SaaS subscription from period t_1 to t_2 is

$$U(\theta, q) = \int_{t_1}^{t_2} (\theta \cdot q(t) + \lambda \cdot q_1) dt. \tag{2b}$$

Equations (2a) and (2b) are generalizations of the commonly used utility function $U(\theta, q) = \theta \cdot q$, which can be obtained from Equations (2a) and (2b) by setting $\lambda = 0$. This allows for two components in the utility function. The $(\theta \cdot q)$ component is standard in vertical models of quality and models consumers as heterogeneous in the benefit they receive from increasing quality. The second component $(\lambda \cdot q_1)$ is a benefit that all buyers receive equally. This is often the case for benefits from basic features, and also, network effects are often modeled as a homogeneous benefit to all users [24]. We use this function to explore the robustness of our insight by establishing limiting cases for our results. Our key findings hold even when this component of the utility function is eliminated (by setting $\lambda = 0$). Consumer type θ is assumed to be uniformly distributed in $[0, 1]$.

We use a multigenerational model that allows for entry of new buyers in period 2. A scaling parameter $\mu \in (0, 1)$ models the size of the market created by the second-

generation users relative to the first generation. By setting $\mu = 0$, we can analyze the case where there is no entry of new buyers in period 2.

The Perpetual Licensing Scheme

WE BEGIN WITH THE CASE WHERE THE SOFTWARE PUBLISHER offers perpetual use licenses. Buyers have several options; they can purchase the perpetual use license in the first period at a price of p_1 and use the product in both periods, thus obtaining a surplus of

$$S_1 = 2\theta(q_1) + 2\lambda(q_1) - p_1.$$

They could purchase in the first period and upgrade at the beginning of the second period at a price of p_u , thus obtaining a total surplus over two periods of

$$S_u = \theta(q_2 + q_1) + 2\lambda(q_1) - p_1 - p_u.$$

The buyers could also decide to forego the software in the first period and purchase it in the second period (S_2) at a price of p_2 , thus earning a surplus of

$$S_2 = \theta(q_2) + \lambda(q_1) - p_2.$$

The surplus for a new buyer who arrives in period two is

$$S_n = \theta(q_2) + \lambda(q_1) - p_2.$$

We now determine the consumer type that is indifferent between these options. Note that because $q_2 = 2(q_1)$ (Equation (1)), there is no single consumer type indifferent between buying in period 1 only and buying in period 2 only. This can be understood by examining the relevant indifference equation to determine the consumer type that is indifferent between these two options: $S_1 = S_2$. It is easy to see that the θ terms cancel from each side of the equation. This implies that all buyers prefer one or the other of these strategies. Thus, only one of these strategies will be pursued by buyers (either $S_1 > S_2$ or vice versa for buyers of all types).

Case 1: $S_2 > S_1$

See Appendix A for solution to this case. We find that the publisher earns greater profits by setting prices that satisfy $S_1 > S_2$ (case 2 below). Thus the seller finds it optimal to set prices so that $S_1 > S_2$. Hence, case 1 is dominated by case 2 and so we focus on case 2 for the remainder of this paper.

Case 2: $S_1 > S_2$

Now we examine the dominant case where the publisher sets prices such that $S_1 > S_2$. In this case, some of the buyers who arrive in period 1 will buy in period 1 and some of those who buy in period 1 will upgrade in period 2. Some of the consumers who arrive in period 2 will buy in period 2. Note that we apply Equation (1) to replace $2q_1$ for q_2 in the profit and indifference equations below.

Let θ_u be the buyer type who is indifferent between the upgrade strategy (buying in period 1 and upgrading in period 2) and purchasing in period 1 only. Therefore, we can obtain θ_u by solving the following indifference equation: $S_u = S_1$. Solving for the indifferent type (θ), we obtain

$$\theta_u = p_u/(q_2 - q_1). \quad (3)$$

Let θ_1 be the buyer type who is indifferent between buying in period 1 only and not buying at all. The relevant indifference equation is $S_1 = 0$. Solving for the indifferent type (θ), we obtain

$$\theta_1 = (p_1/(2q_1)) - \lambda. \quad (4)$$

Let θ_n be the buyer type who arrives in period 2 and is indifferent between buying in period 2 and not buying. The relevant indifference equation is $S_n = 0$. Solving for the indifferent type (θ), we obtain

$$\theta_n = (p_2 - \lambda \cdot q_1)/q_2. \quad (5)$$

The constraints on the indifference points are $0 \leq \theta_1 \leq \theta_u \leq 1$ and $0 \leq \theta_n \leq 1$.

The profit function for perpetual licensing can be stated as

$$\pi_p = (1 - \theta_u)(p_1 + p_u) + (\theta_u - \theta_1)(p_1) + \mu(1 - \theta_n)(p_2) - c(q_1)^2.$$

We replace q_2 , θ_u , θ_1 , θ_n (Equations (1), (3), (4), and (5)) into the profit function and compute the following first derivatives:

$$\delta\pi_p/\delta p_u = 1 - (2p_u/q_1)$$

$$\delta\pi_p/\delta p_1 = 1 + \lambda - (p_1/q_1)$$

$$\delta\pi_p/\delta p_2 = (\mu/2)(2 - (2p_2/q_1) + \lambda)$$

$$\delta\pi_p/\delta q_1 = ((p_1^2 + \mu p_2^2 + 2p_u^2)/(2q_1^2)) - 2c \cdot q_1.$$

Setting these derivatives equal to zero, we solve the first-order conditions to obtain optimal prices and quality:

$$p_1^* = (1 + \lambda)(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)/16c$$

$$p_u^* = (6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)/32c$$

$$p_2^* = (2 + \lambda)(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)/32c$$

$$q_1^* = (6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)/(16c),$$

We calculate the optimal indifference points and find that all constraints are satisfied. The optimal solution is stated below in terms of the first-period quality q_1^* .

Proposition 1: A monopolist seller using perpetual licensing schemes will find it optimal to set the following prices: $p_u^ = q_1^*/2$; $p_1^* = (1 + \lambda)q_1^*$; $p_2^* = ((2 + \lambda)q_1^*)/2$. The optimal quality $q_1^* = [6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2]/(16c)$. The resulting indifference points are $\theta_u^* = 1/2$; $\theta_1^* = (1 - \lambda)/2$; $\theta_n^* = (2 - \lambda)/4$. The seller earns a profit $\pi_p^* = q_1^*[6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2 - 8c \cdot q_1^*]/8$.*

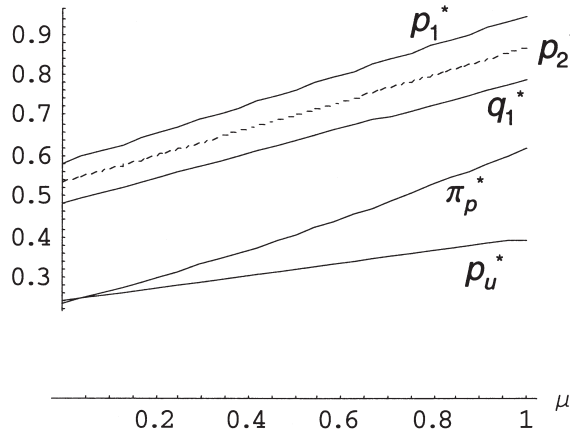


Figure 2. Plot of π_p , q_1 , p_1 , p_2 , and p_u , with $c = 1$, $\lambda = 0.2$

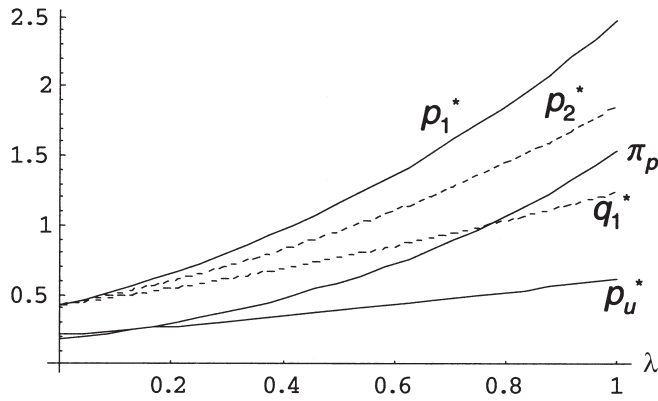


Figure 3. Plot of π_p , q_1 , p_1 , p_2 , and p_u , with $c = 1$, $\mu = 0.2$

We have verified that second-order conditions are satisfied. Proposition 1 shows that more than half of the consumers who arrive in period 1 buy in period 1. A majority of these consumers upgrade in period 2. A majority of consumers that arrive in the second period purchase the software in the second period. The optimal solution is plotted in Figures 2 and 3.

Now we analyze the comparative statics for this solution.

Proposition 2: The optimal profit from perpetual licensing (π_p^) and the optimal quality (q_1^*) and (q_2^*) increase with increase in the proportion of new consumers arriving in period 2 (μ) and the utility from basic features (λ). The optimal profit and quality decrease with the cost of quality (c).*

See Appendix B for proof.

As the number of consumers entering in the second period increases, the software publisher’s marginal revenue from quality improvement increases; thus, the publisher

finds it optimal to invest more in quality. Increase in utility from basic features (λ) has a similar effect.

Subscription Licensing (SaaS)

NOW WE ANALYZE THE CASE WHERE THE SOFTWARE PUBLISHER offers subscription licensing. In each of the two periods, consumers can either subscribe or not subscribe. If a consumer subscribes in period 1 at a price p_1 , he or she is able to use the software throughout period 1. As described above, one of the key advantages of the subscription scheme is that consumers can obtain the benefits of software development (such as new features) faster. Thus subscribers experience regular software upgrades with quality tracking the product development process in period 1: $q \in [q_1, q_2]$. The subscriber's surplus can be computed using Equation (2b) by integrating utility from the period $t_1 = 1$ to $t_2 = 2$:

$$S_1 = \theta((q_1 + q_2)/2) + \lambda(q_1) - p_1.$$

The seller could continue to invest in increasing the quality of the software in period 2; however, as previously explained, in order to maintain comparability in terms of the eventual quality produced under the two licensing schemes, we limit product development time under SaaS so that it is the same as under perpetual licensing. Note that removing this restriction on the seller of the subscription software would result in an even higher quality from the subscription licensing model.

A consumer who subscribes in period 2 at a price of p_2 obtains a surplus of

$$S_2 = \theta(q_2) + \lambda(q_1) - p_2.$$

We now determine the consumer type that is indifferent between subscribing in period 1 and not subscribing. The relevant indifference equation is $S_1 = 0$. Solving for the indifferent θ , we obtain

$$\theta_1 = [2(p_1 - \lambda \cdot q_1)/(q_1 + q_2)]. \quad (6)$$

Let θ_2 be the buyer type who is indifferent between subscribing in period 2 and not subscribing. The relevant indifference equation is $S_2 = 0$. Solving for the indifferent type (θ), we obtain

$$\theta_2 = (p_2 - \lambda \cdot q_1)/q_2. \quad (7)$$

The constraints on the indifference points are $0 \leq \theta_1 \leq 1$ and $0 \leq \theta_2 \leq 1$.

The profit function from perpetual licensing can be stated as

$$\pi_s = (1 - \theta_1)(p_1) + (1 - \theta_2)(p_2)(1 + \mu) - \omega \cdot c(q_1)^2.$$

We replace θ_1 , θ_2 , and q_2 (Equations (1), (6), and (7)) into the profit function above and compute the following first derivatives:

$$\begin{aligned} \delta\pi_s/\delta p_1 &= 1 + (2\lambda/3) - (4p_1/3q_1) \\ \delta\pi_s/\delta p_2 &= (1 + \mu)(q_1(2 + \lambda) - 2p_2)/(2q_1) \end{aligned}$$

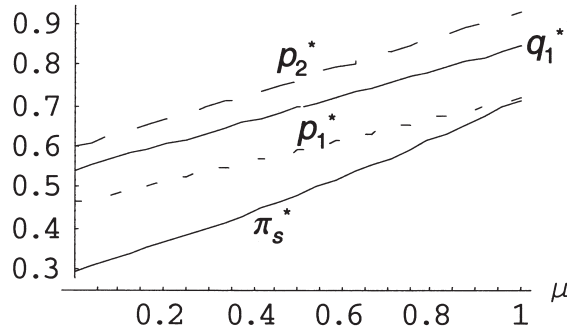


Figure 4. Plot of π_s , q_1 , p_1 , and p_2 , with $c = 1$, $\omega = 1$, $\lambda = 0.2$

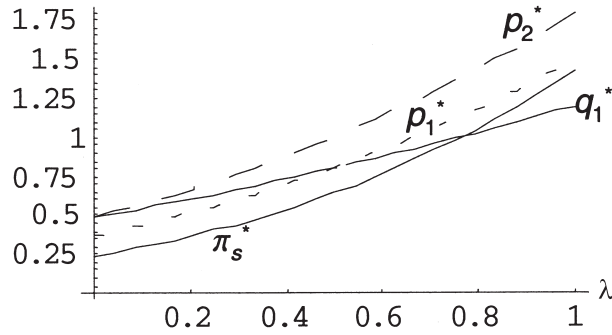


Figure 5. Plot of π_s , q_1 , p_1 , and p_2 , with $c = 1$, $\omega = 1$, $\mu = 0.2$

$$\delta\pi_s/\delta q_1 = (4p_1^2 + 3(1 + \mu)p_2^2 - 12\omega \cdot c \cdot q_1^3)/(6q_1^2).$$

Setting these derivatives equal to zero, we solve the first-order conditions to obtain optimal prices, quality, indifference points, and profits. We find that all constraints are satisfied. The following proposition expresses the optimal solution in terms of the optimal quality (q_1^*):

Proposition 3: A seller using SaaS will find it optimal to set the following prices: $p_1^ = (3 + 2\lambda)q_1^*/4$; $p_2^* = ((2 + \lambda)q_1^*)/2$. The optimal quality is $q_1^* = [21 + 3\mu(2 + \lambda)^2 + \lambda(24 + 7\lambda)]/(48\omega \cdot c)$. The resulting indifference points are $\theta_1^* = (3 - 2\lambda)/6$; $\theta_2^* = (2 - \lambda)/4$. The seller earns a profit of $\pi_s^* = q_1^*[21 + 3\mu(2 + \lambda)^2 + \lambda(24 + 7\lambda) - 24\omega \cdot c \cdot q_1^*/24$.*

Thus more than 50 percent of consumers subscribe in each period. Comparing θ_1^* with θ_2^* , we find that $\theta_2^* - \theta_1^* = \lambda/12$. Hence a smaller fraction of consumers subscribe in period 2 relative to period 1. The fraction of consumers arriving in period 2 who subscribe is the same as the fraction of consumers who arrive in period 1 and subscribe in period 2. The optimal solution is plotted in Figures 4 and 5.

It is interesting to compare the results of Proposition 3 to those in Proposition 1. Note that in the first period, the perpetual licensing scheme results in more users relative to

SaaS. In the second period, the perpetual licensing scheme has greater total number of users but the subscription license has a larger number of users working with the most current version of the product. This occurs because some of the users who buy a perpetual license in period 1 choose not to upgrade in period 2.

Now we analyze the comparative statics for this solution.

Proposition 4: The optimal profit from SaaS (π_s^) and the optimal quality (q_1^* and q_2^*) increase with increase in the proportion of new consumers arriving in period 2 (μ) and the utility from basic features (λ). The optimal profit and quality decrease with the cost of quality (c and ω).*

See Appendix B for proof.

Increasing the number of consumers entering in the second period (μ) raises the software publisher's marginal revenue from quality improvement, thus finding it optimal to invest more in quality. Increase in utility from basic features (λ) has a similar effect on the publisher's incentive to invest in improving product quality.

Comparison of Perpetual and Subscription Licensing

NOW WE COMPARE THE SOLUTIONS OF THE PERPETUAL and subscription licensing schemes. We use q_p to represent the optimal quality under perpetual licensing (q_1) from Proposition 1 and, similarly, q_s for optimal quality under SaaS (q_1 in Proposition 3). Let q_Δ be the difference between the product quality under subscription (q_s) and perpetual licensing (q_p).

$$q_\Delta = q_s - q_p = ((21 + 24\lambda + 7\lambda^2 + 3(2 + \lambda)^2\mu)^2 - 3\omega(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2))/(48\omega \cdot c).$$

Proposition 5: When the utility from basic features $\lambda < (3/5)^{1/2} \approx 0.775$, and the cost of quality is the same for SaaS and perpetual licensing ($\omega = 1$), then the seller invests greater effort in product development resulting in higher quality under subscription licensing relative to perpetual licensing with the difference in product development costs being equal to $((3 - 5\lambda^2)(39 + 48\lambda + 19\lambda^2 + 6\mu(2 + \lambda)^2))/(2304c)$.

See Appendix B for proof.

Proposition 5 indicates that the faster revelation of new product features under subscription licensing provides incentives to the seller that result in greater investment in product development and hence a higher-quality product relative to perpetual licensing. The difference in the number of users between perpetual and SaaS also affects the publisher's incentive to invest in quality. Proposition 5 shows that for the commonly used utility function $U(\theta, q) = \theta \cdot q$, SaaS yields higher equilibrium quality provided that the cost of quality is equal for both licensing methods. Even when $\lambda > 0$, for most reasonable values of λ , SaaS continues to yield higher quality.

Thus we have shown that licensing schemes affect optimal quality and that this effect generally favors SaaS licensing.

Now we compare the optimal profits under the two licensing schemes. Let π_Δ be the difference between profits from subscription and from perpetual licensing: $\pi_\Delta = \pi_s - \pi_p$.

Proposition 6: Comparison of profit under perpetual and SaaS licensing models:

(a) When the utility from basic features $\lambda < (3/5)^{1/2} \approx 0.775$ and the cost of quality is the same for SaaS and perpetual licensing ($\omega = 1$), then the seller earns greater profits from SaaS relative to perpetual licensing. The difference in profits is $\pi_\Delta = [(3 - 5\lambda^2)(39 + 48\lambda + 19\lambda^2 + 6(2 + \lambda)^2\mu)]/(2304c)$.

(b) When $\lambda < (3/5)^{1/2}$, π_Δ increases with μ and decreases otherwise. When either (i) $\lambda > 0.282503$ or (ii) $\lambda \geq 0.268142$ and $\mu > [(36 - \lambda(69 + 5\lambda(36 + 19\lambda)))/(3(2 + \lambda)(10\lambda(1 + \lambda) - 3))]$, π_Δ decreases with λ and increases otherwise.

See Appendix B for proof.

Proposition 6 shows that for the commonly used utility function $U(\theta, q) = \theta \cdot q$, SaaS yields higher profits when the cost of quality for SaaS is no greater than the cost of quality for perpetual licensing. When $\lambda > 0$, for most reasonable values, SaaS continues to yield higher profit. The comparative static for π_Δ is nonmonotonic with respect to μ and λ .

We now examine the impact of the cost of quality for SaaS relative to perpetual licensing.

Proposition 7: When the cost of quality for SaaS relative to the cost of quality for perpetual licensing $\omega < \hat{\omega}_1$, then equilibrium quality is greater under SaaS licensing than under perpetual licensing. When $\omega < \hat{\omega}_2$, then profits are greater under SaaS licensing compared to profits under the perpetual licensing scheme:

$$\hat{\omega}_1 = 1 + \frac{3 - 5\lambda^2}{6(3 + 2\lambda(2 + \lambda)) + 3\mu(2 + \lambda^2)}$$

$$\hat{\omega}_2 = \frac{(21 + 24\lambda + 7\lambda^2 + 3\mu(2 + \lambda)^2)^2}{9(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)^2}$$

See Appendix B for proof.

Proposition 7 shows that in most cases, SaaS yields higher quality relative to perpetual licensing (see Figure 6). Consistent with our findings in Proposition 5, quality is higher under SaaS relative to perpetual licensing when λ is small enough and the cost of quality for software under SaaS is not too much greater than the cost of quality

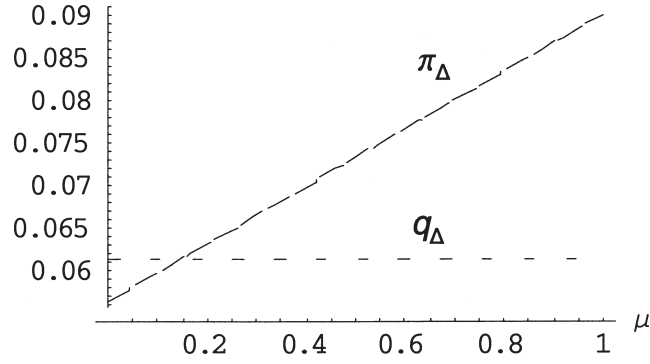


Figure 6. Plot of π_Δ and q_Δ , with $c = 1$, $\lambda = 0.1$, $\omega = 1$

for perpetual licensing. In particular, for the special case of $\lambda = 0$, the utility function reduces to $U = \theta \cdot q$ and $\hat{\omega}_1$ and $\hat{\omega}_2$ are strictly greater than one:

$$\hat{\omega}_1 = 1 + \frac{1}{2(3+2\mu)} > 1$$

and

$$\hat{\omega}_2 = 1 + \frac{13+8\mu}{4(3+2\mu)^2} > 1.$$

We now examine the social welfare generated under perpetual licensing (SW_p) and SaaS (SW_s). These can be obtained by applying the optimal solutions in Propositions 1 and 3, respectively, to the following equations:

$$\begin{aligned} SW_p &= \left(2 \int_{\theta_1}^{\theta_u} (\theta q_p + \lambda q_p) d\theta \right) + \left(\int_{\theta_u}^1 (\theta q_p + 2\theta q_p + 2\lambda q_p) d\theta \right) \\ &\quad + \left(\mu \int_{\theta_n}^1 (2\theta q_p + \lambda q_p) d\theta \right) - cq_p^2 \\ \Rightarrow SW_p &= \frac{(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)^2}{128c} \end{aligned}$$

$$\begin{aligned} SW_s &= \left(\int_{\theta_1}^1 ((3/2)\theta q_s + \lambda q_s) d\theta \right) + \left((1 + \mu) \int_{\theta_2}^1 (2\theta q_s + \lambda q_s) d\theta \right) - \omega \cdot c \cdot q_s^2 \\ \Rightarrow SW_s &= \frac{(21 + 24\lambda + 7\lambda^2 + 3(2 + \lambda)^2 \mu)^2}{1152c \cdot \omega} \end{aligned}$$

Let

$$SW_{\Delta} = SW_s - SW_p$$

$$= \frac{\left(21 + 24\lambda + 7\lambda^2 + 3\mu(2 + \lambda)^2\right)^2 - 9\omega\left(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2\right)^2}{1152c \cdot \omega}.$$

We examine social welfare under the two licensing models and the comparative statics for social welfare in the following proposition:

Proposition 8: Comparison of social welfare generated under perpetual licensing and SaaS:

(a) *When the cost of quality for SaaS relative to the cost of quality for perpetual licensing $\omega < \hat{\omega}_2$, then social welfare is higher under SaaS compared to perpetual licensing.*

(b) *When the cost of quality is the same for SaaS and perpetual licensing ($\omega = 1$) and the benefit from basic features (λ) satisfies $\lambda < (3/5)^{1/2}$, then (i) SaaS generates greater social welfare than perpetual licensing, (ii) the difference between social welfare from SaaS relative to perpetual licensing (SW_{Δ}) increases with the proportion of second-generation consumers (μ), and (iii) SW_{Δ} decreases with the cost of quality (c).*

(c) *The difference between social welfare from SaaS and perpetual licensing (SW_{Δ}) at $\omega = 1$ increases with the utility from basic features (λ) when either (i) $\lambda < 0.268142$ or (ii) $0.268142 \leq \lambda \leq 0.28250$ and $\mu < \hat{\mu}$. Otherwise, SW_{Δ} decreases with λ .*

$$\hat{\omega}_2 = \frac{\left(21 + 24\lambda + 7\lambda^2 + 3\mu(2 + \lambda)^2\right)^2}{9\left(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2\right)^2}$$

$$\hat{\mu} = \frac{36 - \lambda(69 + 5\lambda(36 + 19\lambda))}{3(2 + \lambda)(10\lambda(1 + \lambda) - 3)}.$$

See Appendix B for proof.

Propositions 5, 6, 7, and 8 establish conditions under which SaaS yields higher quality, profits, and social welfare as compared to perpetual licensing (see Figure 7). The difference in optimal investment in quality under the two licensing schemes is stated in Proposition 5. The comparative statics for the difference in optimal investment in software quality is similar to the difference in social welfare as presented in Proposition 8. Our findings show that a monopoly software publisher would prefer SaaS to perpetual licensing when the utility from basic features (λ) is not too large and when the cost of quality for SaaS is not too large relative to the cost of quality

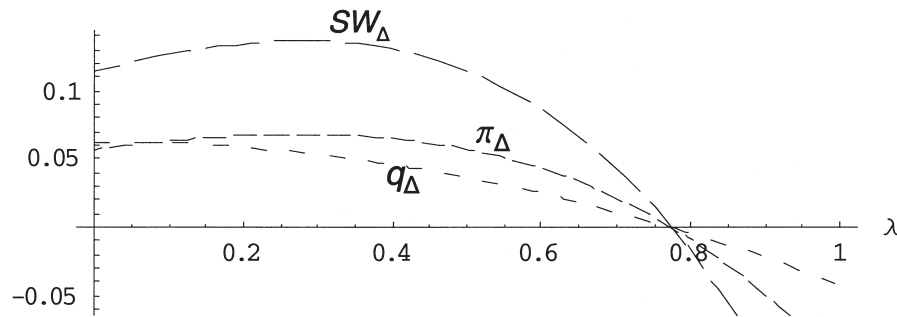


Figure 7. Plot of π_Δ , q_Δ , and SW_Δ , with $c = 1$, $\mu = 0.2$, $\omega = 1$

for perpetual licensing. While an increase in the number of consumers arriving in the second period (μ) generally ($\lambda < (3/5)^{1/2}$) increases the profit advantage of SaaS, the impact of λ depends on certain conditions as stated in Proposition 6.

Discussion and Conclusion

WHAT IS THE IMPACT OF DIFFERENT LICENSING SCHEMES on the publisher's incentive to invest in software quality? Prior academic literature on licensing schemes treated software quality as exogenously determined and instead focused on determining the optimal pricing strategy and a comparison of resulting profits from different licensing schemes. In this paper, we formulate a model of endogenous software quality under the perpetual licensing scheme and under subscription licensing (SaaS). Our model captures key differences in terms of how new software features are disseminated in the perpetual licensing model and the SaaS licensing model. We show that a publisher who adopts the SaaS licensing scheme will typically invest more in software quality compared to a publisher who adopts the perpetual licensing scheme. Despite this higher investment in software quality, the SaaS publisher earns larger profits than a publisher who adopts perpetual licensing. The increased investment in software quality yields higher software quality and higher social welfare under SaaS. These results hold when the cost of quality for SaaS is not much greater than that for perpetual licensing and the utility from basic features (λ) is not too large. Thus for the case of the commonly used utility function ($U(\theta, q) = \theta \cdot q$), the publisher will invest more in software development under SaaS relative to perpetual licensing even when the cost of quality improvement for SaaS is somewhat larger than the cost of quality improvement for perpetual licensing. These incentive effects are unique to SaaS and do not arise in the ASP model because the software publisher provides upgrades to the user or ASP in the same way as in the perpetual licensing model.

Our results have implications for software firms and users as they try to understand the differences between SaaS and perpetual licensing: (1) SaaS involves small, stable cash flows compared to large, irregular payments under perpetual licensing. (2) SaaS users do not need to invest as much in hardware and specialized IT personnel. (3) The

SaaS publisher installs and maintains the application, and can take advantage of his application expertise in these processes. SaaS is also likely to strengthen the publisher's incentive to improve the deployment and maintenance processes. (4) SaaS alters the relative bargaining power between buyers and sellers as the buyers no longer incur large sunk costs. (5) Because application data are stored at the vendor's site, there are implications for security, trust, and switching costs. (6) Large IT projects are risky and exhibit a high failure rate. SaaS can reduce the execution risk for buyers and allow buyers to exit from poor investments with a smaller loss. This paper contributes to the literature on SaaS by showing that SaaS is also likely to lead to a higher rate of improvement in software quality. The SaaS model lends itself to faster time to market for new features, and this, in turn, enhances the publisher's incentive to invest in software quality.

This research can be extended both analytically and empirically. SaaS raises many issues as discussed in the previous paragraph. Future analytical work can examine shifts in bargaining power, economies of scale, and impact on the level of risk in large IT projects. Empirical methods can be used to compare the investment in software development, maintenance costs, prices, publisher profits, and buyer surplus between SaaS and perpetual licensing using data from industry. User surveys can also be used to assess differences in quality between SaaS and perpetual licensing.

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NOTES

1. 2007 Microsoft Office system pricing (<http://office.microsoft.com/en-us/products/FX101754511033.aspx>).

2. Interdependencies between certain features may require that they be released together, thus potentially causing some delays.

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Appendix A: Analysis of Perpetual Licensing Scheme

Case 1: $S_2 > S_1$

WE CONSIDER THE CASE WHERE THE FIRM SETS prices such that $S_2 > S_1$. Note that since $S_2 > S_1$, all consumers prefer buying in period 2 only to buying in period 1 only (without upgrading in period 2). Some consumers may find it optimal to buy in period 1 and upgrade in period 2. Since $S_u - S_2$ is increasing in θ , we know that higher θ -type consumers prefer S_u (buy in period 1 and upgrade in period 2) to S_2 (buy only in period 2). Thus the profit function for the publisher can be formulated as

$$\pi_p = (1 - \theta_u)(p_1 + p_u) + (\theta_u - \theta_2)(p_2) + \mu(1 - \theta_n)(p_2) - c(q_1)^2.$$

We can now determine the consumer type (θ_u) indifferent between the upgrade strategy (buying in period 1 and upgrading in period 2) and buying in period 2 only:

$$\theta_u = (p_1 + p_u - p_2 - \lambda q_1)/q_1.$$

And the consumer type (θ_2) indifferent between buying in period 2 and not buying at all:

$$\theta_2 = (p_2 - \lambda q_1)/(2q_1).$$

And the indifferent consumer type from the second generation who arrives in period 2 and is indifferent between buying in period 2 and not buying at all:

$$\theta_n = (p_2 - \lambda q_1)/(2q_1)$$

The following constraints apply to these indifference points:

$$0 \leq \theta_2 \leq \theta_u \leq 1$$

and

$$0 \leq \theta_n \leq 1.$$

We replace these indifference points in the profit function and set $p_b = p_1 + p_u$ since all buyers who purchase in period 1 also upgrade in period 2. We compute first derivatives of profit with respect to prices:

$$\delta\pi_p/\delta p_b = 1 + \lambda + (2(p_2 - p_b))/(q_1)$$

$$\delta\pi_p/\delta p_2 = (4p_b - \lambda q_1 + (2 + \lambda)\mu q_1 - 2p_2(\mu + 3))/(2q_1)$$

$$\delta\pi_p/\delta q_1 = (2p_b^2 - 4p_2 p_b - 4c q_1^3 + (3 + \mu)p_2^2)/(2q_1^2).$$

Solving the first-order conditions $\delta\pi_p/\delta p_b = 0$, $\delta\pi_p/\delta p_2 = 0$, and $\delta\pi_p/\delta q_1 = 0$, we obtain the following solution:

$$\begin{aligned}\hat{p}_b &= 3q_1/2 + \hat{q}_1\lambda \\ \hat{p}_2 &= q_1 + (\lambda\hat{q}_1)/2 \\ \hat{q}_1 &= (6 + 8\lambda + 3\lambda^2 + \mu(2 + \lambda)^2)/(16c).\end{aligned}$$

And computing optimal indifference points, we find that $\hat{\theta}_u = (1 - \lambda)/2$, $\hat{\theta}_2 = (2 - \lambda)/4$. This violates the constraint $\hat{\theta}_2 \leq \hat{\theta}_u$ because $\hat{\theta}_u - \hat{\theta}_2 = -(\lambda/4)$. Hence, there are no first-generation users who buy only in the second period. Other constraints are satisfied, thus we can modify the profit function accordingly:

$$\pi_p = (1 - \theta_u)(p_1 + p_u) + \mu(1 - \theta_n)(p_2) - c(q_1)^2.$$

Again, replace $p_b = p_1 + p_u$. The indifference points θ_u is obtained by solving $S_u = 0$, which yields $\theta_u = (p_b - 2q_1\lambda)/(3q_1)$ and θ_n is unchanged. We replace these indifference points in the profit function. Computing partial derivatives, we obtain first-order conditions:

$$\begin{aligned}\delta\pi_p/\delta p_b &= 1 + 2\lambda/3 - (2p_b/3q_1) = 0 \\ \delta\pi_p/\delta p_2 &= \mu(1 + (\lambda/2) - (p_2/q_1)) = 0 \\ \delta\pi_p/\delta q_1 &= (2p_b^2 - 12cq_1^3 + 3p_2^2\mu)/(6q_1^2).\end{aligned}$$

Solving these first-order conditions, we obtain a profit of

$$\hat{\pi}_p = (2(3 + 2\lambda)^2 + 3\mu(2 + \lambda)^2)/2304c.$$

The optimal profit from case 2 ($S_1 > S_2$) can be obtained from Proposition 1:

$$\begin{aligned}\pi_p^* &= \frac{(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)^2}{256c} \\ \pi_p^* - \hat{\pi}_p &= \frac{\lambda^2(2(9 + \lambda(12 + 5\lambda)) + 3\mu(2 + \lambda)^2)}{288c} > 0.\end{aligned}$$

Thus the seller will choose prices such that $S_1 > S_2$ and the optimal solution is obtained from solving case 2 where $S_1 > S_2$.

Appendix B: Proof of Propositions

Proof of Proposition 2

$$\begin{aligned}\delta\pi_p^*/\delta\mu &= [(2 + \lambda)^2(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)]/(128c) > 0 \\ \delta\pi_p^*/\delta\lambda &= [(2(2 + \mu) + \lambda(4 + \mu))(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)]/(64c) > 0 \\ \delta\pi_p^*/\delta c &= -[(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)]^2/(256c^2) < 0\end{aligned}$$

$$\delta q_1^*/\delta \mu = [(2 + \lambda)^2]/(16c) > 0$$

$$\delta q_1^*/\delta \lambda = [4 + 2\mu + 4\lambda + \mu\lambda]/(8c) > 0$$

$$\delta q_1^*/\delta c = -[(6 + 4\lambda(2 + \lambda) + \mu(2 + \lambda)^2)]/(16c^2) < 0.$$

Q.E.D.

Proof of Proposition 4

$$\delta \pi_s^*/\delta \mu = [(2 + \lambda)^2(21 + 24\lambda + 7\lambda^2 + 3\mu(2 + \lambda)^2)]/(384\omega \cdot c)$$

$$\rightarrow \delta \pi_s^*/\delta \mu > 0$$

$$\delta \pi_s^*/\delta \lambda = [(12 + 7\lambda + 3(2 + \lambda)\mu)(21 + 24\lambda + 7\lambda^2 + 3\mu(2 + \lambda)^2)]/(576\omega \cdot c)$$

$$\rightarrow \delta \pi_s^*/\delta \lambda > 0$$

$$\delta \pi_s^*/\delta c = -[(21 + 24\lambda + 7\lambda^2 + 3\mu(2 + \lambda)^2)]^2/(2304\omega \cdot c^2)$$

$$\rightarrow \delta \pi_s^*/\delta c < 0$$

$$\delta \pi_s^*/\delta \omega = -[(21 + 24\lambda + 7\lambda^2 + 3\mu(2 + \lambda)^2)]^2/(2304\omega^2 \cdot c)$$

$$\rightarrow \delta \pi_s^*/\delta \omega < 0$$

$$\delta q_1^*/\delta \mu = [(2 + \lambda)^2]/(16\omega \cdot c) > 0$$

$$\delta q_1^*/\delta \lambda = [12 + 7\lambda + 3\mu(2 + \lambda)]/(24\omega \cdot c) > 0$$

$$\delta q_1^*/\delta c = -[(21 + \lambda(24 + 7\lambda) + 3\mu(2 + \lambda)^2)]/(48\omega \cdot c^2) < 0$$

$$\delta q_1^*/\delta \omega = -[(21 + \lambda(24 + 7\lambda) + 3\mu(2 + \lambda)^2)]/(48\omega^2 \cdot c) < 0.$$

Q.E.D.

Proof of Proposition 5

Set $\omega = 1$ in q_Δ . This yields q_Δ (at $\omega = 1$) = $(3 - 5\lambda^2)/(48c)$.

Solving $(3 - 5\lambda^2)/(48c) = 0$ for λ , yields one positive root: $\lambda = (3/5)^{1/2}$.

To obtain the difference in fixed product development costs, we calculate $c(q_s^2 - q_p^2)$ at $\omega = 1$. This yields $((3 - 5\lambda^2)(39 + 48\lambda + 19\lambda^2 + 6\mu(2 + \lambda)^2))/(2304c)$. Q.E.D.

Proof of Proposition 6

(a) At $\omega = 1$, $\pi_\Delta = [(3 - 5\lambda^2)(39 + 48\lambda + 19\lambda^2 + 6(2 + \lambda)^2\mu)]/(2304c)$. Solving $\pi_\Delta = 0$ for λ , we get $\lambda = (3/5)^{1/2}$ as the only feasible root. A quick inspection of π_Δ shows that it is positive when $\lambda < (3/5)^{1/2}$.

(b) $\delta\pi_{\Delta}/\delta\mu = (2 + \lambda)^2(3 - 5\lambda^2)/384$, which is clearly positive when $\lambda < (3/5)^{1/2}$.

$$\delta\pi_{\Delta}/\delta\lambda = (18(2 + \mu) - \lambda(69 + 51\mu + 5\lambda(36 + 19\lambda + 6\mu(3 + \lambda))))/576.$$

Solving $\delta\pi_{\Delta}/\delta\lambda = 0$ for μ , we get

$$\hat{\mu} = \frac{(36 - \lambda(69 + 5\lambda(36 + 19\lambda)))}{(3(2 + \lambda)(10\lambda(1 + \lambda) - 3))}.$$

$\hat{\mu}$ is feasible ($0 \leq \hat{\mu} \leq 1$) only in the region: $0.268142 \leq \lambda \leq 0.282503$.

It can be verified that $\delta\pi_{\Delta}/\delta\lambda < 0$ at $\lambda = 1$; $\delta\pi_{\Delta}/\delta\lambda < 0$ at $\lambda = 0.28$ for $\mu > \hat{\mu}$. Further, $\delta\pi_{\Delta}/\delta\lambda > 0$ at $\lambda = 0$; $\delta\pi_{\Delta}/\delta\lambda > 0$ at $\lambda = 0.28$ for $\mu < \hat{\mu}$. Q.E.D.

Proof of Proposition 7

The condition for comparing quality under SaaS to perpetual licensing in Proposition 7 can be obtained by solving for the value of ω that solves $q_{\Delta} = 0$. Similarly, the condition for comparing profits under SaaS and perpetual licensing can be obtained by solving for the value of ω that solves $\pi_{\Delta} = 0$. It can be verified that both $\hat{\omega}_1$ and $\hat{\omega}_2$ are positive within the feasible region for μ and λ . Q.E.D.

Proof of Proposition 8

(a) Solving $SW_{\Delta} = 0$ for ω yields $\hat{\omega}_2$. It can be verified that $\hat{\omega}_2$ is positive in the feasible region.

(b) The remaining results in Proposition 8 apply to SW_{Δ} evaluated at $\omega = 1$:

$$SW_{\Delta}(\omega = 1) = SW_{\Delta 1} = \frac{(3 - 5\lambda^2)(39 + 48\lambda + 19\lambda^2 + 6\mu(2 + \lambda)^2)}{1152c}.$$

(b)(i) Solving $SW_{\Delta 1} = 0$ for λ , we find a unique feasible root at $\lambda = (3/5)^{1/2}$. $SW_{\Delta 1} > 0$ for $\lambda = 0$, and $SW_{\Delta 1} < 0$ for $\lambda = 1$.

(b)(ii) To see that $SW_{\Delta 1}$ increases with μ when $\lambda < (3/5)^{1/2}$, we compute $\delta SW_{\Delta 1}/\delta\mu$.

$$\delta SW_{\Delta 1}/\delta\mu = (2 + \lambda)^2(3 - 5\lambda^2)/192c.$$

It is easy to verify that this derivative is positive for $\lambda < (3/5)^{1/2}$ and negative otherwise.

(b)(iii) To see that $SW_{\Delta 1}$ decreases with c when $\lambda < (3/5)^{1/2}$, we compute $\delta SW_{\Delta 1}/\delta c$.

$$\delta SW_{\Delta 1} / \delta c = -\frac{(3 - 5\lambda^2)(39 + 48\lambda + 19\lambda^2 + 6\mu(2 + \lambda)^2)}{1152c^2}.$$

It is easy to verify that this derivative is negative for $\lambda < (3/5)^{1/2}$ and positive otherwise.

(c) To understand the comparative static of SW_{Δ_1} with respect to λ , we compute $\delta SW_{\Delta_1}/\delta\lambda$.

$$\frac{\partial SW_{\Delta_1}}{\partial\lambda} = \frac{18(2+\mu) - \lambda(69 + 51\mu + 5\lambda(36 + 19\lambda + 6\mu(3+\lambda)))}{288c}.$$

Solving $\delta SW_{\Delta_1}/\delta\lambda = 0$ for μ , we obtain

$$\hat{\mu} = \frac{36 - \lambda(69 + 5\lambda(36 + 19\lambda))}{3(2 + \lambda)(10\lambda(1 + \lambda) - 3)}.$$

$\hat{\mu}$ is feasible ($0 \leq \hat{\mu} \leq 1$) when $0.268142 \leq \lambda \leq 0.282503$. It is easy to verify that $\delta SW_{\Delta_1}/\delta\lambda > 0$ at $\lambda = 0$ and $\delta SW_{\Delta_1}/\delta\lambda < 0$ at $\lambda = 1$. Further, $\delta SW_{\Delta_1}/\delta\lambda > 0$ at $\lambda = 0.28$, and $\mu < \hat{\mu}$ and $\delta SW_{\Delta_1}/\delta\lambda < 0$ at $\lambda = 0.28$, and $\mu > \hat{\mu}$. Hence proved. Q.E.D.

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